

$X(1835)$ and $\eta(1760)$ observed by the BES Collaboration

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Abstract

With the assumption that the $X(1835)$ and $\eta(1760)$ recently observed by the BES Collaboration are the 3^1S_0 meson states, the strong decays of these two states are investigated in the 3P_0 decay model. We find that the predicted total widths of the $X(1835)$ and $\eta(1760)$ can be reasonably reproduced with the $X(1835) - \eta(1760)$ mixing angle lying on the range from -0.26 to $+0.55$ radians. Further, the mixing angle of the $X(1835)$ and $\eta(1760)$ is phenomenologically determined to be about -0.24 radians in the presence of the $\pi(1800)$, $K(1830)$, $\eta(1760)$ and $X(1835)$ belonging to the 3^1S_0 meson nonet. Our estimated mixing angle can naturally account for the widths of the $X(1835)$ and $\eta(1760)$, which shows that the assignment of the $X(1835)$ and $\eta(1760)$, together the $\pi(1800)$ and $K(1830)$ as the 3^1S_0 $q\bar{q}$ members seems reasonable.

Key words: mesons, 3P_0 model

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1 Introduction

In 2005, the BES collaboration observed a narrow peak called $X(1835)$ in the reaction $J/\psi \rightarrow \gamma\pi^+\pi^-\eta'$ with a statistical significance of 7.7σ , and the mass and width of the $X(1835)$ obtained from the fit with a Breit-Wigner function are $M = (1833.7 \pm 6.1 \pm 2.7)$ MeV and $\Gamma = (67.7 \pm 20.3 \pm 7.7)$ MeV, respectively[1]. No partial wave analysis has been made but based on production and decay, it is most likely that the $J^{PC}I^G$ of the $X(1835)$ is $0^{-+}0^+$.

The BES collaboration suggests that the $X(1835)$ is related to the $p\bar{p}$ mass threshold enhancement observed in the $J/\psi \rightarrow \gamma p\bar{p}$ channel[2], which therefore triggering many exotic speculations of the nature of the $X(1835)$ such as glueball[3, 4, 5], $p\bar{p}$ baryonium[6, 7] etc. It should be noted that there is no strong experimental evidence that the $p\bar{p}$ threshold enhancement and the $X(1835)$ are the same resonance, as pointed out by Huang and Zhu[8]; very probably they have completely different underlying structures. In the presence of the $X(1835)$ being not related to the $p\bar{p}$ mass threshold enhancement, Huang and Zhu suggested the $X(1835)$ is the second radial excitation of the η' [8], while Klempt and Zaitsev rather believed the $X(1835)$ to be the η' 's first radial excitation[9].

The $\eta(1760)$ was reported by the MARKIII Collaboration in radiative J/ψ decays into $\omega\omega$ [10] and $\rho\rho$ [11]. It was also observed by the DM2 Collaboration in J/ψ radiative decays in the $\rho\rho$ [12] decay mode with a mass of $M = (1760 \pm 11)$ MeV and a width of $\Gamma = (60 \pm 16)$ MeV and in the $\omega\omega$ decay mode[13]. Vijande et al. suggested that it can be identified as the 2^1S_0 $q\bar{q}$ state[14]; Page and Li proposed that it could have hybrid admixture[15]. A reanalysis MARKIII data on $J/\psi \rightarrow \gamma 4\pi$ performed by Bugg et al. suggested that the decay mode should not be in the $\rho\rho$ pseudoscalar wave but in the $(\pi\pi)_{\text{S-wave}}(\pi\pi)_{\text{S-wave}} 0^{++}$ scalar wave[16]. This was supported by BES Collaboration suggesting the $\rho\rho$ resonance at 1760 MeV should be interpreted as the scalar meson in its $\sigma\sigma$ decay[17]. The conclusions regarding the presence of this pseudoscalar signal in the MARKIII 4π data become therefore controversial.

More Recently, the decay channel $J/\psi \rightarrow \gamma\omega\omega$, $\omega \rightarrow \pi^+\pi^-\pi^0$ was analyzed by the BES collaboration, and a strong enhancement was found in the $\omega\omega$ invariant mass distribution at 1.76 GeV. The partial wave analysis indicated that the structure was predominantly pseudoscalar

with small scalar and tensor contributions. The mass of the pseudoscalar state also called $\eta(1760)$ is $M = (1744 \pm 10 \pm 15)$ MeV, the width $\Gamma = (244_{-21}^{+24} \pm 25)$ MeV and the product branching fraction is $\text{Br}(J/\psi \rightarrow \gamma\eta(1760))\text{Br}(\eta(1760) \rightarrow \omega\omega) = (1.98 \pm 0.08 \pm 0.32) \times 10^{-3}$ [18]. The BES Collaboration suggested that it could be a mixture of the glueball and the $q\bar{q}$ meson.

From PDG2006[19], the 1^1S_0 meson nonet (π , η , η' and K) as well as the 2^1S_0 members [$\pi(1300)$, $\eta(1295)$ and $\eta(1475)$] have been well established. We argue that the $X(1835)$ and $\eta(1760)$ reported by BES Collaboration can be assigned as the ordinary 3^1S_0 $q\bar{q}$ states. The purpose of this work is to check in the simple picture of the $X(1835)$ and $\eta(1760)$ as the 3^1S_0 $q\bar{q}$ states, whether the total widths of these two states can be reasonably reproduced in the 3P_0 meson decay model or not.

The organization of this paper is as follows. In section 2, the brief review of the 3P_0 decay model is given (For the detailed review see e.g. Refs.[20, 21, 22, 23].) In section 3, the decay widths of the $X(1835)$ and $\eta(1760)$ are presented. The mixing angle of the $X(1835) - \eta(1760)$ is given in section 4, and a summary and conclusion are given in section 5.

2 The 3P_0 meson decay model

The 3P_0 decay model, also known as the quark-pair creation model, was originally introduced by Micu[24] and further developed by Le Yaouanc et al.[20]. The 3P_0 decay model has been widely used to evaluate the strong decays of hadrons[25, 26, 27, 28, 29, 30, 31, 32, 33, 34], since it gives a good description of many of the observed decay amplitudes and partial widths of the hadrons. The main assumption of the 3P_0 decay model is that strong decays take place via the creation of a 3P_0 quark-antiquark pair from the vacuum. The new produced quark-antiquark pair, together with the $q\bar{q}$ within the initial meson regroups into two outgoing mesons in all possible quark rearrangement ways, which corresponds to the two decay diagrams as shown in Fig.1 for the meson decay process $A \rightarrow B + C$.

The transition operator T of the decay $A \rightarrow BC$ in the 3P_0 model is given by

$$T = -3\gamma \sum_m \langle 1m1 - m | 00 \rangle \int d^3\vec{p}_3 d^3\vec{p}_4 \delta^3(\vec{p}_3 + \vec{p}_4) \mathcal{Y}_1^m\left(\frac{\vec{p}_3 - \vec{p}_4}{2}\right) \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_3^\dagger(\vec{p}_3) d_4^\dagger(\vec{p}_4), \quad (1)$$

where γ is a dimensionless parameter representing the probability of the quark-antiquark pair

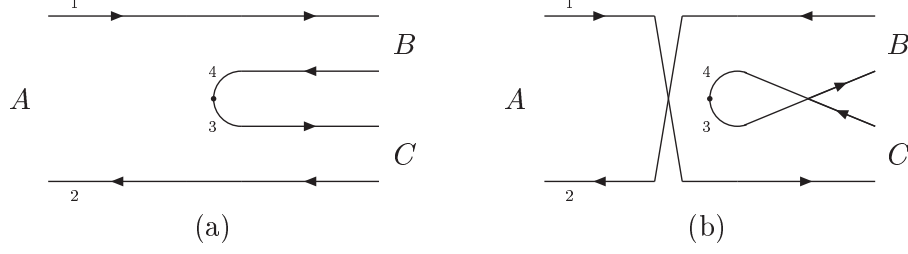


Figure 1: The two possible diagrams contributing to $A \rightarrow B + C$ in the 3P_0 model.

$q_3\bar{q}_4$ with $J^{PC} = 0^{++}$ creation from the vacuum, \vec{p}_3 and \vec{p}_4 are the momenta of the created quark q_3 and antiquark \bar{q}_4 , respectively. ϕ_0^{34} , ω_0^{34} , and $\chi_{1,-m}^{34}$ are the flavor, color, and spin wave functions of the $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\mathcal{Y}_1^m(\vec{p}) \equiv |p|Y_1^m(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\bar{q}_4$.

For the meson wave function, we adopt the mock meson $|A(n_A^{2S_A+1}L_A J_A M_{J_A})(\vec{P}_A)\rangle$ defined by[35]

$$\begin{aligned}
|A(n_A^{2S_A+1}L_A J_A M_{J_A})(\vec{P}_A)\rangle &\equiv \sqrt{2E_A} \sum_{M_{L_A}, M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle \\
&\times \int d^3\vec{p}_A \psi_{n_A L_A M_{L_A}}(\vec{p}_A) \chi_{S_A M_{S_A}}^{12} \phi_A^{12} \omega_A^{12} \\
&\times |q_1(\frac{m_1}{m_1+m_2}\vec{P}_A + \vec{p}_A) \bar{q}_2(\frac{m_2}{m_1+m_2}\vec{P}_A - \vec{p}_A)\rangle, \quad (2)
\end{aligned}$$

where m_1 and m_2 are the masses of the quark q_1 with a momentum of \vec{p}_1 and the antiquark \bar{q}_2 with a momentum of \vec{p}_2 , respectively. n_A is the radial quantum number of the meson A composed of $q_1\bar{q}_2$. $\vec{S}_A = \vec{s}_{q_1} + \vec{s}_{q_2}$, $\vec{J}_A = \vec{L}_A + \vec{S}_A$, \vec{s}_{q_1} (\vec{s}_{q_2}) is the spin of q_1 (q_2), \vec{L}_A is the relative orbital angular momentum between q_1 and q_2 . $\vec{P}_A = \vec{p}_1 + \vec{p}_2$, $\vec{p}_A = \frac{m_1\vec{p}_1 - m_1\vec{p}_2}{m_1+m_2}$. $\langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle$ is a Clebsch-Gordan coefficient, and E_A is the total energy of the meson A . $\chi_{S_A M_{S_A}}^{12}$, ϕ_A^{12} , ω_A^{12} , and $\psi_{n_A L_A M_{L_A}}(\vec{p}_A)$ are the spin, flavor, color, and space wave functions of the meson A , respectively. The mock meson satisfies the normalization condition

$$\langle A(n_A^{2S_A+1}L_A J_A M_{J_A})(\vec{P}_A) | A(n_A^{2S_A+1}L_A J_A M_{J_A})(\vec{P}'_A) \rangle = 2E_A \delta^3(\vec{P}_A - \vec{P}'_A). \quad (3)$$

The S -matrix of the process $A \rightarrow BC$ is defined by

$$\langle BC | S | A \rangle = I - 2\pi i \delta(E_A - E_B - E_C) \langle BC | T | A \rangle, \quad (4)$$

with

$$\langle BC|T|A\rangle = \delta^3(\vec{P}_A - \vec{P}_B - \vec{P}_C) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}, \quad (5)$$

where $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$ is the helicity amplitude of $A \rightarrow BC$. In the center of mass frame of meson A , $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$ can be written as

$$\begin{aligned} \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\vec{P}) &= \gamma \sqrt{8E_A E_B E_C} \sum_{\substack{M_{L_A}, M_{S_A}, \\ M_{L_B}, M_{S_B}, \\ M_{L_C}, M_{S_C}, m}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle \\ &\times \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B} \rangle \langle L_C M_{L_C} S_C M_{S_C} | J_C M_{J_C} \rangle \\ &\times \langle 1m1 - m | 00 \rangle \langle \chi_{S_B M_{S_B}}^{14} \chi_{S_C M_{S_C}}^{32} | \chi_{S_A M_{S_A}}^{12} \chi_{1-m}^{34} \rangle \\ &\times [f_1 I(\vec{P}, m_1, m_2, m_3) + (-1)^{1+S_A+S_B+S_C} f_2 I(-\vec{P}, m_2, m_1, m_3)], \quad (6) \end{aligned}$$

with $f_1 = \langle \phi_B^{14} \phi_C^{32} | \phi_A^{12} \phi_0^{34} \rangle$ and $f_2 = \langle \phi_B^{32} \phi_C^{14} | \phi_A^{12} \phi_0^{34} \rangle$, corresponding to the contributions from Figs. 1 (a) and 1 (b), respectively, and

$$\begin{aligned} I(\vec{P}, m_1, m_2, m_3) &= \int d^3 \vec{p} \psi_{n_B L_B M_{L_B}}^* \left(\frac{m_3}{m_1+m_2} \vec{P}_B + \vec{p} \right) \psi_{n_C L_C M_{L_C}}^* \left(\frac{m_3}{m_2+m_3} \vec{P}_B + \vec{p} \right) \\ &\times \psi_{n_A L_A M_{L_A}}(\vec{P}_B + \vec{p}) \mathcal{Y}_1^m(\vec{p}), \quad (7) \end{aligned}$$

where $\vec{P} = \vec{P}_B = -\vec{P}_C$, $\vec{p} = \vec{p}_3$, m_3 is the mass of the created quark q_3 .

The spin overlap in terms of Winger's $9j$ symbol can be given by

$$\begin{aligned} &\langle \chi_{S_B M_{S_B}}^{14} \chi_{S_C M_{S_C}}^{32} | \chi_{S_A M_{S_A}}^{12} \chi_{1-m}^{34} \rangle = \\ &\sum_{S, M_S} \langle S_B M_{S_B} S_C M_{S_C} | S M_S \rangle \langle S_A M_{S_A} 1 - m | S M_S \rangle \\ &(-1)^{S_C+1} \sqrt{3(2S_A+1)(2S_B+1)(2S_C+1)} \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & S_A \\ \frac{1}{2} & \frac{1}{2} & 1 \\ S_B & S_C & S \end{array} \right\}. \quad (8) \end{aligned}$$

In order to compare with experiment conventionally, $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\vec{P})$ can be converted into the partial amplitude by a recoupling calculation[36]

$$\begin{aligned} \mathcal{M}^{LS}(\vec{P}) &= \sum_{\substack{M_{J_B}, M_{J_C}, \\ M_S, M_L}} \langle L M_L S M_S | J_A M_{J_A} \rangle \langle J_B M_{J_B} J_C M_{J_C} | S M_S \rangle \\ &\times \int d\Omega Y_{LM_L}^* \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\vec{P}). \quad (9) \end{aligned}$$

If we consider the relativistic phase space, the decay width $\Gamma(A \rightarrow BC)$ in terms of the partial wave amplitudes is

$$\Gamma(A \rightarrow BC) = \frac{\pi P}{4M_A^2} \sum_{LS} |\mathcal{M}^{LS}|^2. \quad (10)$$

Here $P = |\vec{P}| = \frac{\sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]}}{2M_A}$, M_A , M_B , and M_C are the masses of the meson A , B , and C , respectively.

The decay width can be derived analytically if the simple harmonic oscillator (SHO) approximation for the meson space wave functions is used. In momentum-space, the SHO wave function is

$$\psi_{nLM_L}(\vec{p}) = R_{nL}^{\text{SHO}}(p) Y_{LM_L}(\Omega_p), \quad (11)$$

where the radial wave function is given by

$$R_{nL}^{\text{SHO}} = \frac{(-1)^n (-i)^L}{\beta^{\frac{3}{2}}} \sqrt{\frac{2n!}{\Gamma(n + L + \frac{3}{2})}} \left(\frac{p}{\beta}\right)^L e^{-\frac{p^2}{2\beta^2}} L_n^{L+\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right). \quad (12)$$

Here β is the SHO wave function scale parameter, and $L_n^{L+\frac{1}{2}}(\frac{p^2}{\beta^2})$ is an associated Laguerre polynomial.

The SHO wave functions can not be regarded as realistic, however, they are a *de facto* standard for many nonrelativistic quark model calculations. Moreover, the more realistic space wave functions such as those obtained from Coulomb plus the linear potential model do not always result in systematic improvements due to the inherent uncertainties of the 3P_0 decay model itself[26, 27, 29]. The SHO wave function approximation is commonly employed in the 3P_0 decay model in literature. In the present work, the SHO wave function approximation for the meson space wave functions is taken.

3 Decays of the $X(1835)$ and $\eta(1760)$ in the 3P_0 model

It is well known that in a meson nonet, the physical isoscalar states can mix. With the assumption that the $X(1835)$ and $\eta(1760)$ being the 3^1S_0 meson nonet, the $X(1835) - \eta(1760)$ mixing can be parameterized as

$$\eta(1760) = \cos \phi n\bar{n} - \sin \phi s\bar{s}, \quad (13)$$

$$X(1835) = \sin \phi n\bar{n} + \cos \phi s\bar{s}, \quad (14)$$

where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ are the pure 3^1S_0 nonstrange and strange states, respectively.

According to (10), the partial widths of $X(1835)$ and $\eta(1760)$ become with mixing

$$\Gamma(\eta(1760) \rightarrow BC) = \frac{\pi P}{4M_{\eta(1760)}^2} \sum_{LS} |\cos \phi \mathcal{M}_{n\bar{n} \rightarrow BC}^{LS} - \sin \phi \mathcal{M}_{s\bar{s} \rightarrow BC}^{LS}|^2, \quad (15)$$

$$\Gamma(X(1835) \rightarrow BC) = \frac{\pi P}{4M_{X(1835)}^2} \sum_{LS} |\sin \phi \mathcal{M}_{n\bar{n} \rightarrow BC}^{LS} + \cos \phi \mathcal{M}_{s\bar{s} \rightarrow BC}^{LS}|^2. \quad (16)$$

Under the SHO wave function approximation, the parameters used in the $3P_0$ decay model involve the $q\bar{q}$ pair production strength parameter γ , the SHO wave function scale parameter β , and the masses of the constituent quarks. In the present work, we take $\gamma = 6.95$ and $\beta = 0.4$ GeV, the typical values used to evaluate the light meson decays[27, 28, 29, 30, 31, 32]¹, and $m_u = m_d = 0.33$ GeV, $m_s = 0.55$ GeV[33]. Based on the partial wave amplitudes of the 3^1S_0 $q\bar{q}$ into two other mesons listed in the Appendix A and the flavor and charge multiplicity factors shown in Table 2, from (15) and (16), the numerical values of the partial decay widths of the $\eta(1760)$ and $X(1835)$ are listed in Table 1. Masses of the final mesons are taken from PDG2006[19]. We assume the $a_0(1450)$ is the ground scalar mesons as Ref.[31, 32]. The total widths of the $X(1835)$ and $\eta(1760)$ are shown in Fig. 2 as functions of the mixing angle ϕ .

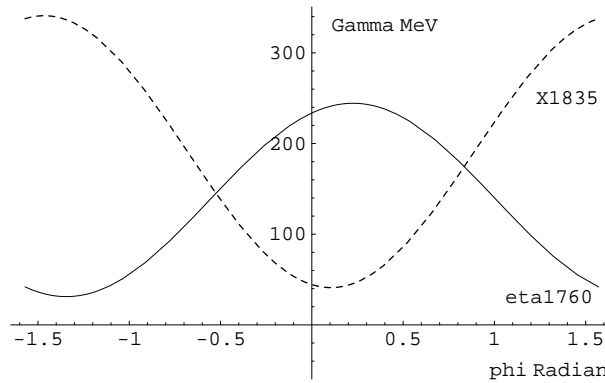


Figure 2: Theoretical total widths of the $X(1835)$ and $\eta(1760)$ versus the mixing angle ϕ

From Fig. 2, we find that in the presence of the $X(1835) - \eta(1760)$ mixing angle ϕ lying on the range from about -0.26 to $+0.55$ radians, both $\Gamma(X(1835))$ and $\Gamma(\eta(1760))$ can be

¹Our value of γ is higher than that used by other groups such as [29, 30, 31, 32] by a factor of $\sqrt{96\pi}$ due to different field conventions, constant factor in T , etc. The calculated results of the widths are, of course, unaffected.

Table 1: Decays of the $X(1835)$ and $\eta(1760)$ as the 3^1S_0 $q\bar{q}$ in the 3P_0 model. $c \equiv \cos \phi$, $s \equiv \sin \phi$.

	$\eta(1760)$	$X(1835)$
Mode	$\Gamma_i(\text{MeV})$	$\Gamma_i(\text{MeV})$
$\rho\rho$	$92.7c^2$	$117.1s^2$
$\omega\omega$	$29.4c^2$	$38.7s^2$
$a_2(1320)\pi$	$33.1c^2$	$82.3s^2$
$a_0(1450)\pi$	$26.7c^2$	$30.0s^2$
KK^*	$51.8c^2 + 93.1cs + 41.8s^2$	$27.8c^2 - 83.6cs + 62.8s^2$
K^*K^*		$16.6c^2 + 21.3cs + 6.8s^2$
	$\Gamma = 233.7c^2 + 93.1cs + 41.8s^2$	$\Gamma = 44.4c^2 - 62.3cs + 337.7s^2$
	$\Gamma_{\text{expt}} = 244^{+24}_{-21} \pm 25$	$\Gamma_{\text{expt}} = 67.7 \pm 20.3 \pm 7.7$

reasonably reproduced. In order to check whether the possibility of $-0.26 \leq \phi \leq +0.55$ radians exists or not, below we shall estimate the $X(1835)$ - $\eta(1760)$ mixing angle phenomenologically.

4 The $X(1835)$ - $\eta(1760)$ mixing angle

In the $n\bar{n}$ and $s\bar{s}$ basis, the mass-squared matrix describing the $X(1835)$ and $\eta(1760)$ mixing can be written as[37, 38]

$$M^2 = \begin{pmatrix} M_{n\bar{n}}^2 + 2A_m & \sqrt{2}A_m X \\ \sqrt{2}A_m X & M_{s\bar{s}}^2 + A_m X^2 \end{pmatrix}, \quad (17)$$

where $M_{n\bar{n}}$ and $M_{s\bar{s}}$ are the masses of the states $n\bar{n}$ and $s\bar{s}$, respectively, A_m denotes the total annihilation strength of the $q\bar{q}$ pair for the light flavors u and d , X describes the $SU(3)$ -breaking ratio of the nonstrange and strange quark propagators via the constituent quark mass ratio m_u/m_s . The masses of the two physical states $\eta(1760)$ and $X(1835)$ can be related to the matrix M^2 by the unitary matrix $U = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}$

$$UM^2U^\dagger = \begin{pmatrix} M_{\eta(1760)}^2 & 0 \\ 0 & M_{X(1835)}^2 \end{pmatrix}. \quad (18)$$

$n\bar{n}$ is the orthogonal partner of $\pi(3^1S_0)$, the isovector state of 3^1S_0 meson nonet, and one can expect that $n\bar{n}$ degenerates with $\pi(3^1S_0)$ in effective quark masses, here we take $M_{n\bar{n}} = M_{\pi(3^1S_0)} = M_{\pi(1800)}$ ². From the masses of the constituent quarks used to evaluate the widths of the $\eta(1760)$ and $X(1835)$ in section 3, we have $X = 0.33/0.55 = 0.6$. Inputting the masses of the $\pi(1800)$, $\eta(1760)$ and $X(1835)$, with the help of the Gell-Mann-Okubo mass formula $M_{s\bar{s}}^2 = 2M_{K(3^1S_0)}^2 - M_{n\bar{n}}^2$ [40], from relation (18) we can have $A_m = -0.09 \text{ GeV}^2$ and $M_{K(3^1S_0)} = 1.82 \text{ GeV}$ in agreement with the experimental result $M_{K(1830)} \sim 1830 \text{ MeV}$ [19]. The predicted total width $\Gamma_{K(3^1S_0)}$ given by the 3P_0 decay model is about 201 MeV [32], also roughly compatible with the experimental result $\Gamma(1830) \sim 250 \text{ MeV}$ [19]. Therefore, in the presence of the $\pi(1800)$, $\eta(1760)$ and $X(1835)$ being the 3^1S_0 states, the $K(1830)$ seems an excellent candidate for the second radial excitation of the Kaon[39].

Based on the values of the above parameters involved in (17), we have

$$U = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} = \begin{pmatrix} +0.97 & +0.24 \\ -0.24 & +0.97 \end{pmatrix}, \quad (19)$$

which gives $\phi = -0.24$ radians, just lying on the range of about $-0.26 \sim +0.55$ radians. From Table 1, this estimated mixing angle leads to $\Gamma(1760) = 200.6 \text{ MeV}$ and $\Gamma(1835) = 75.7 \text{ MeV}$, both in agreement with the experimental results within errors. This shows that in the presence of the $\pi(1800)$, $K(1830)$, $X(1835)$ and $\eta(1760)$ belonging to the 3^1S_0 meson nonet, the total widths of the $X(1835)$ and $\eta(1760)$ can be naturally accounted for in the 3P_0 decay model. Therefore, the assignment of the $X(1835)$ and $\eta(1760)$, together the $\pi(1800)$ and $K(1830)$ as the 3^1S_0 $q\bar{q}$ members seems reasonable.

As mentioned in section 1, in J/ψ radiative decays, the DM2 Collaboration observed the $\rho\rho$ signal with a mass of $(1760 \pm 11) \text{ MeV}$ and a width of $\Gamma = (60 \pm 16) \text{ MeV}$ [12], while the BES Collaboration found the $\omega\omega$ signal with a mass of $(1744 \pm 10 \pm 15) \text{ MeV}$ and a width of $\Gamma = (244_{-21}^{+24} \pm 25) \text{ MeV}$ [18]. It is suggested that the $\rho\rho$ signal is compatible with the $\omega\omega$ signal[9, 19]; we regard them as incompatible. The 3P_0 decay model predicts that $\frac{\mathcal{B}(\eta(1760) \rightarrow \rho\rho)}{\mathcal{B}(\eta(1760) \rightarrow \omega\omega)} = 3$, i.e.,

²The nature of the $\pi(1800)$ is controversial, different interpretations such as a 3^1S_0 $q\bar{q}$ [$\pi(3^1S_0)$] and hybrid (π_H) exist[9]. There is the possibility that the two states, $\pi(3^1S_0)$ and π_H have been observed in the 1800 MeV mass region, as pointed out by[31]. Here we consider the $\pi(1800)$ as the $\pi(3^1S_0)$, as suggested by PDG2002[39]

the $\rho\rho$ yield should be 3 times larger than that for $\omega\omega$, incompatible with the measured yields $(1.44 \pm 0.12 \pm 0.21) \times 10^{-3}$ for $\rho\rho$ [12] and $(1.98 \pm 0.08 \pm 0.32) \times 10^{-3}$ for $\omega\omega$ [18], which indicates that the $\rho\rho$ signal at 1760 MeV[12] may be incompatible with the $\omega\omega$ signal at 1760 MeV[18], assuming that the 3P_0 decay model is accurate.

Table 1 and (19) show that the modes K^*K and K^*K^* are the dominant decay modes of the $X(1835)$. These two modes are experimental attractive because their $\cos\phi \sin\phi$ cross terms have opposite signs, so the ratio $\mathcal{B}(X(1835) \rightarrow K^*K)/\mathcal{B}(X(1835) \rightarrow K^*K^*)$ depends strongly on the mixing angle ϕ .

5 Summary and conclusion

Assuming that the $X(1835)$ and $\eta(1760)$ reported by the BES Collaboration are the ordinary 3^1S_0 $q\bar{q}$ states, we evaluate the strong decays of these two states in the framework of the 3P_0 meson decay model. We find that when the $X(1835) - \eta(1760)$ mixing angle lies on the range from -0.26 to $+0.55$ radians, both the total width of the $X(1835)$ and that of the $\eta(1760)$ can be reasonably reproduced. Also, in the presence of the $\pi(1800)$, $\eta(1760)$ and $X(1835)$ belonging to the 3^1S_0 meson nonet, the $K(1830)$ seems an excellent candidate for the 3^1S_0 kaon, and the $X(1835) - \eta(1760)$ mixing angle of about -0.24 radians can be phenomenologically obtained, naturally accounting for the total widths of the $X(1835)$ and $\eta(1760)$. We therefore conclude that the assignment of the $X(1835)$ and $\eta(1760)$, together the $\pi(1800)$ and $K(1830)$ as the 3^1S_0 $q\bar{q}$ members seems reasonable, and the $X(1835)$ is mostly strange while the $\eta(1760)$ is mainly non-strange. Also, we suggest the $\rho\rho$ signal at 1760 MeV reported by the DM2 Collaboration[12] may be incompatible with the $\omega\omega$ signal at 1760 MeV observed by the BES Collaboration.

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Appendix A: The amplitudes for the 3^1S_0 $q\bar{q}$ decay in 3P_0 model

$$\begin{aligned}
\mathcal{M}^{LS}(3^1S_0 \rightarrow 1^3S_1 + 1^3S_1) = & \\
-2\gamma_e & - \frac{[m_1 m_2 (m_2 - m_3) m_3 + m_2^2 m_3^2 + m_1^2 (m_2^2 + m_2 m_3 + m_3^2)] P^2}{3\beta^2 (m_1 + m_3)^2 (m_2 + m_3)^2} \sqrt{E_a E_b E_c} \frac{1}{\pi^{3/4}} (f_1 + f_2) P \\
& \times \left[45\beta^4 (m_1 + m_3)^4 (m_2 + m_3)^4 (19m_1 m_2 + 14m_1 m_3 + 14m_2 m_3 + 9m_3^2) - 12\beta^2 (m_1 + m_3)^2 \right. \\
& \quad \times (m_2 + m_3)^2 (13m_1 m_2 + 14m_1 m_3 + 14m_2 m_3 + 15m_3^2) (m_2 m_3 + 2m_1 m_2 + m_1 m_3)^2 P^2
\end{aligned}$$

$$+4(m_2 m_3 + 2m_1 m_2 + m_1 m_3)^4 (m_1 m_2 + 2m_1 m_3 + 2m_2 m_3 + 3m_3^2) P^4 \Big] \\ \times \frac{1}{2187\sqrt{5}\beta^{11/2}} \frac{1}{(m_1 + m_3)^5 (m_2 + m_3)^5} \quad (\text{A.1})$$

$$\mathcal{M}^{LS}(3^1 S_0 \rightarrow 1^3 S_1 + 1^1 S_0) = \\ -\sqrt{\frac{2}{5}} \gamma e \frac{[m_1 m_2 (m_2 - m_3) m_3 + m_2^2 m_3^2 + m_1^2 (m_2^2 + m_2 m_3 + m_3^2)] P^2}{3\beta^2 (m_1 + m_3)^2 (m_2 + m_3)^2} \sqrt{E_a E_b E_c} \frac{1}{\pi^{3/4}} (f_1 - f_2) P \\ \times \Big[45\beta^4 (m_1 + m_3)^4 (m_2 + m_3)^4 (19m_1 m_2 + 14m_1 m_3 + 14m_2 m_3 + 9m_3^2) - 12\beta^2 (m_1 + m_3)^2 \\ \times (m_2 + m_3)^2 (13m_1 m_2 + 14m_1 m_3 + 14m_2 m_3 + 15m_3^2) (m_2 m_3 + 2m_1 m_2 + m_1 m_3)^2 P^2 \\ + 4(m_2 m_3 + 2m_1 m_2 + m_1 m_3)^4 (m_1 m_2 + 2m_1 m_3 + 2m_2 m_3 + 3m_3^2) P^4 \Big] \\ \times \frac{1}{2187\beta^{11/2}} \frac{1}{(m_1 + m_3)^5 (m_2 + m_3)^5} \quad (\text{A.2})$$

$$\mathcal{M}^{LS}(3^1 S_0 \rightarrow 1^3 P_0 + 1^1 S_0) = 2i\gamma e \frac{-(m_1 m_2 (m_2 - m_3) m_3 + m_2^2 m_3^2 + m_1^2 (m_2^2 + m_2 m_3 + m_3^2)) P^2}{3\beta^2 (m_1 + m_3)^2 (m_2 + m_3)^2} \sqrt{E_a E_b E_c} \frac{1}{\pi^{3/4}} \\ \times \left\{ -(f_1 + f_2) \frac{5\sqrt{5}}{81\sqrt{3}\beta^{1/2}} \right. \\ + \Big[(16m_2^2 m_3^2 + 14m_2 m_3^3 + 17m_1^2 m_2^2 + 17m_1^2 m_2 m_3 + 2m_1^2 m_3^2 + 36m_1 m_2^2 m_3 + 37m_1 m_2 m_3^2 + 5m_1 m_3^3) f_2 \\ + (2m_2^2 m_3^2 + 5m_2 m_3^3 + 17m_1 m_2^2 m_3 + 37m_1 m_2 m_3^2 + 14m_1 m_3^3 + 17m_1^2 m_2^2 + 36m_1^2 m_2 m_3 + 16m_1^2 m_3^2) f_1 \Big] \\ \times \frac{\sqrt{5}}{243\sqrt{3}\beta^{5/2}} \frac{1}{(m_1 + m_3)^2 (m_2 + m_3)^2} P^2 \\ + \Big[(11m_2^2 m_3^2 + 14m_2 m_3^3 + 3m_1^2 m_2^2 + 3m_1^2 m_2 m_3 - 3m_1^2 m_3^2 + 16m_1 m_2^2 m_3 + 21m_1 m_2 m_3^2 - m_1 m_3^3) f_2 \\ + (3m_1^2 m_2^2 + 16m_1^2 m_2 m_3 + 11m_1^2 m_3^2 + 3m_1 m_2^2 m_3 + 21m_1 m_2 m_3^2 + 14m_1 m_3^3 - 3m_2^2 m_3^2 - m_2 m_3^3) f_1 \Big] \\ \times \frac{-4}{729\sqrt{15}\beta^{9/2}} \frac{(m_2 m_3 + 2m_1 m_2 + m_1 m_3)^2}{(m_1 + m_3)^4 (m_2 + m_3)^4} P^4 \\ + [(m_1 m_2 - m_1 m_3 + 2m_2 m_3) f_2 + (m_1 m_2 + 2m_1 m_3 - m_2 m_3) f_1] (m_2 m_3 + 2m_1 m_2 + m_1 m_3)^4 \\ \times (m_1 m_2 + 2m_1 m_3 + 2m_2 m_3 + 3m_3^2) \frac{4}{6561\sqrt{15}\beta^{13/2}} \frac{1}{(m_1 + m_3)^6 (m_2 + m_3)^6} P^6 \Big\} \quad (\text{A.3})$$

$$\mathcal{M}^{LS}(3^1 S_0 \rightarrow 1^3 P_2 + 1^1 S_0) = 2i\gamma e \frac{-(m_1 m_2 (m_2 - m_3) m_3 + m_2^2 m_3^2 + m_1^2 (m_2^2 + m_2 m_3 + m_3^2)) P^2}{3\beta^2 (m_1 + m_3)^2 (m_2 + m_3)^2} \sqrt{E_a E_b E_c} \frac{1}{\pi^{3/4}} \\ \times \left\{ \left[\left(-\frac{11m_1^2}{(m_1 + m_3)^2} + \frac{14m_2(6m_2 + 5m_3)}{(m_2 + m_3)^2} + \frac{m_1(28m_2 + 25m_3)}{(m_1 + m_3)(m_2 + m_3)} \right) f_2 \right. \right. \\ + \left. \left(\frac{14m_1^2}{(m_1 + m_3)^2} + \frac{m_2(14m_2 + 25m_3)}{(m_2 + m_3)^2} + \frac{m_1(73m_2 + 70m_3)}{(m_1 + m_3)(m_2 + m_3)} \right) f_1 \right] \frac{-27\sqrt{2}}{6561\sqrt{15}\beta^{5/2}} P^2 \\ + \Big[(12m_2^2 m_3^2 + 14m_2 m_3^3 + 7m_1^2 m_2^2 + 7m_1^2 m_2 m_3 - 2m_1^2 m_3^2 + 20m_1 m_2^2 m_3 + 23m_1 m_2 m_3^2 - m_1 m_3^3) f_2 \\ + (-2m_2^2 m_3^2 - m_2 m_3^3 + 7m_1^2 m_2^2 + 20m_1^2 m_2 m_3 + 12m_1^2 m_3^2 + 7m_1 m_2^2 m_3 + 23m_1 m_2 m_3^2 + 14m_1 m_3^3) f_1 \Big] \\ \times (m_2 m_3 + 2m_1 m_2 + m_1 m_3)^2 \frac{4\sqrt{2}}{729\sqrt{15}\beta^{9/2}} \frac{1}{(m_1 + m_3)^4 (m_2 + m_3)^4} P^4 \\ + [(m_1 m_2 - m_1 m_3 + 2m_2 m_3) f_2 + (m_1 m_2 + 2m_1 m_3 - m_2 m_3) f_1] (m_2 m_3 + 2m_1 m_2 + m_1 m_3)^4 \\ \times (m_1 m_2 + 2m_1 m_3 + 2m_2 m_3 + 3m_3^2) \frac{-4\sqrt{2}}{6561\sqrt{15}\beta^{13/2}} \frac{1}{(m_1 + m_3)^6 (m_2 + m_3)^6} P^6 \Big\} \quad (\text{A.4})$$

Appendix B: Flavor and Weight factors

The flavor factors f_1 and f_2 can be calculated using the matrix notation introduced in Ref.[22] with the meson flavor wavefunctions following the conventions of Ref.[41] for the special process with definite charges like $n\bar{n} \rightarrow \rho^+ \rho^-$. In order to obtain the general (i.e. charge independent) width of decays like $n\bar{n} \rightarrow \rho\rho$, one should multiply the width $\Gamma(n\bar{n} \rightarrow \rho^+ \rho^-)$ by a charge multiplicity factor \mathcal{F} . The f_1 , f_2 and \mathcal{F} for all the processes considered in this work are given in Table 2.

Table 2: Flavor and charge multiplicity factors

General decay	subprocess	f_1	f_2	\mathcal{F}
$n\bar{n} \rightarrow \rho\rho$	$n\bar{n} \rightarrow \rho^+\rho^-$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$\frac{3}{2}$
$n\bar{n} \rightarrow a_0(1450)\pi$	$n\bar{n} \rightarrow a_0(1450)^+\pi^-$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	3
$n\bar{n} \rightarrow a_2(1320)\pi$	$n\bar{n} \rightarrow a_2(1320)^+\pi^-$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	3
$n\bar{n} \rightarrow \omega\omega$	$n\bar{n} \rightarrow \omega\omega$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{2}$
$n\bar{n} \rightarrow K^*K$	$n\bar{n} \rightarrow K^{*+}K^-$	$-\frac{1}{\sqrt{6}}$	0	4
$n\bar{n} \rightarrow K^*K^*$	$n\bar{n} \rightarrow K^{*+}K^{*-}$	$-\frac{1}{\sqrt{6}}$	0	2
$s\bar{s} \rightarrow K^*K$	$s\bar{s} \rightarrow K^{*+}K^-$	0	$-\frac{1}{\sqrt{3}}$	4
$s\bar{s} \rightarrow K^*K^*$	$s\bar{s} \rightarrow K^{*+}K^{*-}$	0	$-\frac{1}{\sqrt{3}}$	2